

JAIPURIA INSTITUTE OF MANAGEMENT, NOIDA

PGDM (C)

FIRST TRIMESTER (Batch 2023-25) SET 1

END TERM EXAMINATION, OCTOBER 2023

Course Name	SFM	Course Code	20501
Max. Time	2 Hours	Max. Marks	40 MM

INSTRUCTIONS:

- a. Attempt all questions
- b. Use of calculators is allowed

1. The accompanying table shows the incidence of malaria and two other similar illnesses. If a person lives in an area affected by all three diseases, what is the probability that he or she will develop at least one of the three illnesses? (Assume that contracting one disease is an event independent from contracting any other disease) (5 marks)

	Cases per year	Number at Risk (Millions)
Malaria	110 million	2,100
Schistosomiasis	200 million	600
Sleeping sickness	25,000	50

2. Consider a test for an illness. The test has a known reliability:

- a. When administered to an ill person, the test will indicate so with probability 0.92.
- b. When administered to a person who is not ill, the test will erroneously give a positive result with probability 0.04.

Suppose the illness is rare and is known to affect only 0.1% of the entire population. If a person is randomly selected from the entire population and is given the test and the result is positive, calculate the posterior probability (posterior to the test result) that the person is ill? (2.5X2=5 marks)

3. A drug manufacturer believes there is a 0.95 chance that the Food and Drug Administration (FDA) will approve a new drug the company plans to distribute if the results of current testing show that the drug causes no side effects. The manufacturer further believes there is a 0.50 probability that the FDA will approve the drug if the test shows that the drug does cause side effects. A physician working for the drug manufacturer believes there is a 0.20 probability that tests will show that the drug causes

side effects. What is the probability that the drug will be approved by the FDA?

(5 marks)

4. Under the system of floating exchange rates, the rate of foreign money to the U.S. dollar is affected by many random factors, and this leads to the assumption of a normal distribution of small daily fluctuations. The rate of U.S. dollar per euro is believed in April 2007 to have a mean of 1.36 and a standard deviation of 0.03. Calculate the following.

a. The probability that tomorrow's rate will be above 1.42.

b. The probability that tomorrow's rate will be below 1.35.

c. The probability that tomorrow's exchange rate will be between 1.16 and 1.23
(2X3=6 marks)

5. The amount of fuel consumed by the engines of a jetliner on a flight between two cities is a normally distributed random variable X with mean 5.7 tons and standard deviation 0.5. Carrying too much fuel is inefficient as it slows the plane. If, however, too little fuel is loaded on the plane, an emergency landing may be necessary. The airline would like to calculate the amount of fuel to load so that there will be a 0.99 probability that the plane will arrive at its destination. (4 marks)

6. A certain commodity is known to have a price that is stable through time and does not change according to any known trend. Price, however, does change from day to day in a random fashion. If the price is at a certain level one day, it is as likely to be at any level the next day within some probability bounds approximately given by a normal distribution. The mean daily price is believed to be \$14.25. To test the hypothesis that the average price is \$14.25 versus the alternative hypothesis that it is not \$14.25, a random sample of 36 daily prices is collected. The results are Mean \$16.50 and SD \$5.8. Using $\alpha = 0.05$, test the null hypothesis? (5 marks)

7. Three different techniques namely medication, exercises and special diet are randomly assigned to (individuals diagnosed with high blood pressure) lower the blood pressure. After four weeks the reduction in each person's blood pressure is recorded. Test at 5% level, whether there is significant difference in mean reduction of blood pressure among the three techniques. (5 marks)

Medication	10	12	9	15	13
Exercise	6	8	3	0	2
Diet	5	9	12	8	4

8. The heights (in cm.) of a group of fathers and sons are given below

Heights of fathers: 158 166 163 165 167 170 167 172 177 181

Heights of Sons : 163 158 167 170 160 180 170 175 172 175

Estimate the height of son when the height of the father is 164 cm.

(5 marks)

Statistics Formulas

Position of p^{th} percentile = $\left(\frac{n+1}{100}\right)^{\text{P}}$

OR

$$\frac{nP}{100}$$

Interquartile Range = $Q_3 - Q_1$

Range = Highest - Lowest

Mean = $\frac{\sum x_i}{N}$ (sum of all items)

$\mu = \frac{\sum fx}{\sum f}$ (discrete grouped data)

$\mu = \frac{\sum fm}{\sum f}$ (continuous grouped data)

where m = mid-point

Variance & Std. Deviation

Population

Mean = μ

σ^2 (variance) = $\frac{\sum_{i=1}^N (x_i - \mu)^2}{N}$

OR

$$\sigma^2 = \frac{\sum_{i=1}^N x_i^2 - \left(\frac{\sum_{i=1}^N x_i}{N}\right)^2}{N}$$

$$= \frac{\sum_{i=1}^N x_i^2}{N} - \mu^2$$

For discrete/continuous data

$$\begin{aligned}\sigma^2 &= \frac{\sum f(x - \mu)^2}{\sum f} \\ &= \frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2 \\ &= \frac{\sum fx^2}{\sum f} - \mu^2\end{aligned}$$

$\sigma = \sqrt{\sigma^2}$ (std. deviation)

$\sum (x_i - \mu) = 0$

Sample
Mean = $\bar{x} = \frac{\sum_{i=1}^n x_i}{n}$

$$\begin{aligned}s^2 \text{ (variance)} &= \frac{\sum (x_i - \bar{x})^2}{n-1} \\ &= \frac{\sum_{i=1}^n x_i^2 - \left(\frac{\sum_{i=1}^n x_i}{n}\right)^2}{n-1}\end{aligned}$$

For discrete/continuous data

$$\begin{aligned}s^2 &= \frac{\sum f(x - \bar{x})^2}{\sum f - 1} \\ &= \frac{\sum fx^2 - \frac{(\sum fx)^2}{\sum f}}{\sum f - 1}\end{aligned}$$

S (std. deviation) = $\sqrt{s^2}$

Skewness = $\frac{1}{n} \frac{\sum (x_i - \bar{x})^3}{s^3}$

Kurtosis = $\frac{1}{n} \frac{\sum (x_i - \bar{x})^4}{s^4}$

Coefficient of Variation

$$C.V = \frac{\sigma}{\mu} (100)$$

A.P (Arithmetic Progression)

$AP = l = T_n = a + (n-1)d$

$S_n = \frac{n}{2} [2a + (n-1)d] = \frac{n}{2} [a + l]$

Arithmetic Mean (AM) = $\frac{a+c}{2} = b$
where a, b, c are in A.P.

Geometric Progression (G.P)

Ratio (r) = $\frac{2^{\text{nd}} \text{ term}}{1^{\text{st}} \text{ term}}$

$S_n = \frac{a(r^n - 1)}{r - 1}$, where $r > 1$

$S_n = \frac{a(1 - r^n)}{1 - r}$, where $r < 1$

Logarithms

$\log mn = \log m + \log n$

$\log m/n = \log m - \log n$

$\log m^n = n \log m$

Geometric Mean (GM)

If a, b, c are in G.P

$$GM \Rightarrow \frac{b}{a} = \frac{c}{b} \Rightarrow b = \sqrt{ac}$$

• GM = $\sqrt[n]{\text{Product of all } x \text{ values}}$

$$= (x_1 \times x_2 \times x_3 \times \dots \times x_n)^{1/n}$$

$$\log GM = \frac{1}{n} (\log(x_1 \times x_2 \times x_3 \times \dots \times x_n))$$

$$= \frac{1}{n} (\log x_1 + \log x_2 + \log x_3 + \dots)$$

$$= \frac{\sum_{i=1}^n \log x_i}{n}$$

$$\bullet GM = \text{Antilog} \left(\frac{\sum_{i=1}^n \log x_i}{n} \right)$$

• GM for discrete grouped data

$$= AL \left[\frac{\sum_{i=1}^n (f_i \cdot \log x_i)}{\sum f} \right]$$

For continuous grouped data
 $x_i = m_i$ (mid-point)

• Harmonic Progression (H.P)

$$\bullet H.P = \frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d}, \dots, \frac{1}{a+(n-1)d}$$

$$\bullet n^{\text{th}} \text{ term} = \frac{1}{a+(n-1)d}$$

• Harmonic Mean (HM)

where a, b, c are in H.P

$$b = \frac{2ac}{a+c}$$

$$\bullet H.M = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \dots + \frac{1}{x_n}}$$
$$= \frac{n}{\sum \left[\frac{1}{x_i} \right]}$$

• H.M for discrete grouped data

$$= \frac{\sum f}{\sum \left(f_i \times \frac{1}{x_i} \right)}$$

Sets & Probability

Null Set / Empty set / Void Set = ϕ

Universal Set = U

Mutually exclusive / Disjoint Set

$$A \cap B = \phi$$

Independent

Probability of disjoint sets

$$P(A \text{ or } B) = P(A) + P(B)$$

Union

$$\bullet P(A \text{ or } B) = P(A) + P(B) - P(A \cap B)$$
$$= P(A \cup B)$$

$$\bullet P(\text{at least } 1) = 1 - P(\text{none}) = P(A \cup B)$$

$$\bullet P(\phi) = 0$$

• $P(A')$: Complementary Event of A / Not A

$$= P(\bar{A}) = 1 - P(A)$$

Intersection

$$\bullet P(A \cap B) = P(A \& B)$$

$P(A \text{ but not } B) / P(A - B) / P(A \cap B')$

$$P(\text{only } A) : P(A) - P(A \cap B)$$

$P(\text{Exactly one}) :$

$$P(A \cap B') + P(A' \cap B)$$

$$= P(A \cup B) - P(A \cap B)$$

$$P(A' \cap B') / P(\text{None}) : 1 - P(A \cup B)$$

$$P(A' \cup B) = P(B \cap A) + P(A')$$

Permutation & Combination

$$n! = n(n-1)(n-2) \dots 3 \cdot 2 \cdot 1$$

$$0! = 1 = 1!$$

Arrangement: Permutation (P)

$${}^n P_r = \frac{n!}{(n-r)!}$$

selection / chosen / group / combination: Combination (C)

$${}^n C_r = \frac{n!}{r! (n-r)!}$$

General Laws of Multiplication

$$P(X \cap Y) = P(X) \cdot P\left(\frac{Y}{X}\right) = P(Y) \cdot P\left(\frac{X}{Y}\right)$$

2 events are independent

$$\Rightarrow P(A) \times P(B) = P(A \& B)$$

OR

$$P(A \cap B) = P(A) \cdot P(B)$$

$$P(A \text{ or } B \text{ or Both}) = P(A \cup B)$$

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$

$$P(A_1 \cup A_2 \cup A_3 \dots \cup A_n) = 1 - P(\bar{A}_1 \cap \bar{A}_2 \cap \bar{A}_3 \cap \dots \cap \bar{A}_n)$$

Bayes Theorem

$$P\left(\frac{A}{E}\right) = \frac{P\left(\frac{E}{A}\right) \cdot P(A)}{P\left(\frac{E}{A}\right) \cdot P(A) + P\left(\frac{E}{B}\right) \cdot P(B) + P\left(\frac{E}{C}\right) \cdot P(C)}$$

Expected Value

$$\mu = E(X) = \sum_{\text{all } x} [x \cdot P(x)]$$

Expected value of $h(x)$

$$E[h(x)] = \sum_{\text{all } x} [h(x) \cdot P(x)]$$

Variance (σ^2)

$$\sigma^2 = V(X) = E[(X - \mu)^2] \\ = \sum_{\text{all } x} [(x - \mu)^2 \cdot P(x)]$$

OR

$$\sigma^2 = E(X^2) - [E(X)]^2$$

Law of Total Probability

$$P(A) = P\left(\frac{A}{B}\right) \cdot P(B) + P\left(\frac{A}{B^c}\right) \cdot P(B^c)$$

t-test

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

Standardization of variable

Standard Normal Random Variable (Z)

$$Z \sim N(0, 1^2)$$

where mean = 0

std. deviation = 1

$$Z = \frac{x - \mu}{\sigma}$$

Inverse transformation of Z to X

$$X = \mu + \sigma \cdot Z$$

The Central Limit Theorem

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$$

sample size ≥ 30 : Z-statistic

sample size < 30 : t-statistic

Confidence Interval

$$\mu \pm Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

if sampling is done

$$\bar{x} \pm Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

when using t-statistics

degree of freedom (df) = n - 1

$$\bar{x} \pm t_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

Population Proportion

confidence interval for p =

$$\hat{p} \pm Z_{\alpha/2} \sqrt{\frac{\hat{p} \cdot \hat{q}}{n}}$$

where $\hat{q} = (1 - \hat{p})$

Chi-Square [Variance (confidence interval)]

$$\left[\frac{(n-1)s^2}{\chi_{\alpha/2}^2}, \frac{(n-1)s^2}{\chi_{1-\alpha/2}^2} \right] = \chi^2$$

Hypothesis Testing

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

H_0 : Null Hypothesis

H_1 : Alternate Hypothesis (3)

ANOVA $SSB = \left(\sum \frac{T_i^2}{n_i} \right) - \frac{(\sum x)^2}{n}$

$$SSW = \sum x^2 - \left(\sum \frac{T_i^2}{n_i} \right)$$

$$MSB = \frac{SSB}{k-1}, \quad MSW = \frac{SSW}{n-k}$$

$$F = \frac{MSB}{MSW}$$

Regression $\hat{y} = a + bx$

$$a = \bar{y} - b\bar{x}, \quad b = \frac{S_{xy}}{S_{xx}}$$

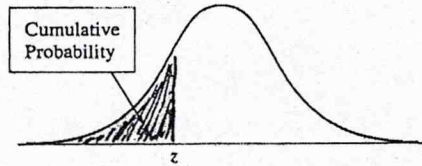
$$S_{xx} = \sum x^2 - \frac{(\sum x)^2}{n}$$

$$S_{xy} = \sum xy - \frac{(\sum x)(\sum y)}{n}$$

F Values for $\alpha = 0.05$

d_2	d_1								
	1	2	3	4	5	6	7	8	9
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5
2	18.51	19.00	19.16	19.25	19.3	19.33	19.35	19.37	19.38
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96
inf	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88

APPENDIX A

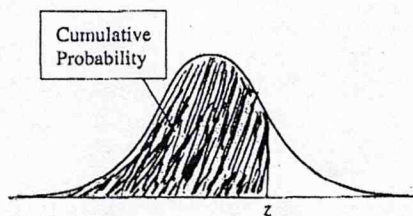


Cumulative probability for z is the area under the standard normal curve to the left of z

TABLE A Standard Normal Cumulative Probabilities

z	.00
-5.0	.000000287
-4.5	.00000340
-4.0	.0000317
-3.5	.000233

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641



Cumulative probability for z is the area under the standard normal curve to the left of z

TABLE A Standard Normal Cumulative Probabilities (continued)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

z	.00
3.5	.999767
4.0	.9999683
4.5	.9999966
5.0	.999999713

df	U.1	U.05	U.025	U.02	U.01	U.005
1	3.078	6.314	12.706	15.895	31.821	63.657
2	1.886	2.920	4.303	4.849	6.965	9.925
3	1.638	2.353	3.182	3.482	4.541	5.841
4	1.533	2.132	2.776	2.999	3.747	4.604
5	1.476	2.015	2.571	2.757	3.365	4.032
6	1.440	1.943	2.447	2.612	3.143	3.707
7	1.415	1.895	2.365	2.517	2.998	3.499
8	1.397	1.860	2.306	2.449	2.896	3.355
9	1.383	1.833	2.262	2.398	2.821	3.250
10	1.372	1.812	2.228	2.359	2.764	3.169
11	1.363	1.796	2.201	2.328	2.718	3.106
12	1.356	1.782	2.179	2.303	2.681	3.055
13	1.350	1.771	2.160	2.282	2.650	3.012
14	1.345	1.761	2.145	2.264	2.624	2.977
15	1.341	1.753	2.131	2.249	2.602	2.947
16	1.337	1.746	2.120	2.235	2.583	2.921
17	1.333	1.740	2.110	2.224	2.567	2.898
18	1.330	1.734	2.101	2.214	2.552	2.878
19	1.328	1.729	2.093	2.205	2.539	2.861
20	1.325	1.725	2.086	2.197	2.528	2.845
21	1.323	1.721	2.080	2.189	2.518	2.831
22	1.321	1.717	2.074	2.183	2.508	2.819
23	1.319	1.714	2.069	2.177	2.500	2.807
24	1.318	1.711	2.064	2.172	2.492	2.797
25	1.316	1.708	2.060	2.167	2.485	2.787
26	1.315	1.706	2.056	2.162	2.479	2.779
27	1.314	1.703	2.052	2.158	2.473	2.771
28	1.313	1.701	2.048	2.154	2.467	2.763
29	1.311	1.699	2.045	2.150	2.462	2.756
30	1.310	1.697	2.042	2.147	2.457	2.750
31	1.309	1.696	2.040	2.144	2.453	2.744
32	1.309	1.694	2.037	2.141	2.449	2.738
33	1.308	1.692	2.035	2.138	2.445	2.733
34	1.307	1.691	2.032	2.136	2.441	2.728
35	1.306	1.690	2.030	2.133	2.438	2.724
36	1.306	1.688	2.028	2.131	2.434	2.719
37	1.305	1.687	2.026	2.129	2.431	2.715
38	1.304	1.686	2.024	2.127	2.429	2.712
39	1.304	1.685	2.023	2.125	2.426	2.708
40	1.303	1.684	2.021	2.123	2.423	2.704
41	1.303	1.683	2.020	2.121	2.421	2.701
42	1.302	1.682	2.018	2.120	2.418	2.698
43	1.302	1.681	2.017	2.118	2.416	2.695
44	1.301	1.680	2.015	2.116	2.414	2.692
45	1.301	1.679	2.014	2.115	2.412	2.690
46	1.300	1.679	2.013	2.114	2.410	2.687
47	1.300	1.678	2.012	2.112	2.408	2.685
48	1.299	1.677	2.011	2.111	2.407	2.682
49	1.299	1.677	2.010	2.110	2.405	2.680
50	1.299	1.676	2.009	2.109	2.403	2.678

df	0.1	0.05	0.025	0.02	0.01	0.005
51	1.298	1.675	2.008	2.108	2.402	2.676
52	1.298	1.675	2.007	2.107	2.400	2.674
53	1.298	1.674	2.006	2.106	2.399	2.672
54	1.297	1.674	2.005	2.105	2.397	2.670
55	1.297	1.673	2.004	2.104	2.396	2.668
56	1.297	1.673	2.003	2.103	2.395	2.667
57	1.297	1.672	2.002	2.102	2.394	2.665
58	1.296	1.672	2.002	2.101	2.392	2.663
59	1.296	1.671	2.001	2.100	2.391	2.662
60	1.296	1.671	2.000	2.099	2.390	2.660
61	1.296	1.670	2.000	2.099	2.389	2.659
62	1.295	1.670	1.999	2.098	2.388	2.657
63	1.295	1.669	1.998	2.097	2.387	2.656
64	1.295	1.669	1.998	2.096	2.386	2.655
65	1.295	1.669	1.997	2.096	2.385	2.654
66	1.295	1.668	1.997	2.095	2.384	2.652
67	1.294	1.668	1.996	2.095	2.383	2.651
68	1.294	1.668	1.995	2.094	2.382	2.650
69	1.294	1.667	1.995	2.093	2.382	2.649
70	1.294	1.667	1.994	2.093	2.381	2.648
71	1.294	1.667	1.994	2.092	2.380	2.647
72	1.293	1.666	1.993	2.092	2.379	2.646
73	1.293	1.666	1.993	2.091	2.379	2.645
74	1.293	1.666	1.993	2.091	2.378	2.644
75	1.293	1.665	1.992	2.090	2.377	2.643
76	1.293	1.665	1.992	2.090	2.376	2.642
77	1.293	1.665	1.991	2.089	2.376	2.641
78	1.292	1.665	1.991	2.089	2.375	2.640
79	1.292	1.664	1.990	2.088	2.374	2.640
80	1.292	1.664	1.990	2.088	2.374	2.639
81	1.292	1.664	1.990	2.087	2.373	2.638
82	1.292	1.664	1.989	2.087	2.373	2.637
83	1.292	1.663	1.989	2.087	2.372	2.636
84	1.292	1.663	1.989	2.086	2.372	2.636
85	1.292	1.663	1.988	2.086	2.371	2.635
86	1.291	1.663	1.988	2.085	2.370	2.634
87	1.291	1.663	1.988	2.085	2.370	2.634
88	1.291	1.662	1.987	2.085	2.369	2.633
89	1.291	1.662	1.987	2.084	2.369	2.632
90	1.291	1.662	1.987	2.084	2.368	2.632
91	1.291	1.662	1.986	2.084	2.368	2.631
92	1.291	1.662	1.986	2.083	2.368	2.630
93	1.291	1.661	1.986	2.083	2.367	2.630
94	1.291	1.661	1.986	2.083	2.367	2.629
95	1.291	1.661	1.985	2.082	2.366	2.629
96	1.290	1.661	1.985	2.082	2.366	2.628
97	1.290	1.661	1.985	2.082	2.365	2.627
98	1.290	1.661	1.984	2.081	2.365	2.627
99	1.290	1.660	1.984	2.081	2.365	2.626
100	1.290	1.660	1.984	2.081	2.364	2.626